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Comparison of centroid and paracone estimates of tree volume[1]

HARRY V. WIANT, JR.

Division of Forestry, West Virginia University, Morgantown, WV 26506, U.S.A.

GEOFFREY B. WOOD

Department of Forestry, Australian National University, GPO Box 4, Canberra, Australia ACT 2601

AND

ROBERT R. FORSLUND

Ontario Ministry of Natural Resources, Ontario Forest Research Institute, Centre for Northern Forest Ecosystem Research, c/o Lakehead University, Thunder Bay, Ont., Canada P7B 5E1

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The use of the centroid and paracone methods for deriving the volume of 957 sample trees from four data sets was compared. The four data sets were as follows: Australian radiata pine (*Pinus radiata* D. Don) (114 trees); six species of Australian commercial hardwoods (140 trees); and two sets of Appalachian hardwoods, one comprising four species (538 trees) and the other 13 species (165 trees). Comparable accuracies of prediction were achieved with both methods, although the paracone method was slightly better for all data sets except radiata pine when predicting total volume. The similarity in the predictions was shown to be a direct result of the similarity in the total volume equations of the two methods. The centroid method consistently predicted merchantable volumes better than the paracone method, which had a tendency to give poorer estimates for smaller stems as the merchantable height decreased.

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Les méthodes du centroïde et du paracône ont été comparées pour calculer le volume de 957 arbres-échantillons provenant de quatre groupes de données, i.e., le pin radiata (*Pinus radiata* D. Don) d'Australie (114 arbres); six espèces commerciales de bois franc australien (140 arbres); et deux groupes de bois franc appalachien comprenant respectivement quatre (538 arbres) et 13 espèces (165 arbres). Les deux méthodes affichent des précisions de prédiction du volume total comparables, bien que la méthode du paracône se soit avérée légèrement supérieure pour tous les groupes de données sauf en ce qui a trait au pin radiata. Il a été démontré que la similarité dans les prédictions découle directement de la similarité des équations du volume total utilisées par les deux méthodes. La méthode du centroïde prédit généralement mieux le volume marchand que la méthode du paracône qui tend à fournir de moins bons estimés pour les petites tiges au fur et à mesure que la hauteur marchande diminue.

[Traduit par la rédaction]

Introduction

Forslund (1982) developed a tree bole model that describes a geometrical form between a paraboloid and a cone, which he called a "paracone." Using a formula [1] derived from the paracone model with $A = 1.5$, the total volume (V_t (m^3)) of a tree bole can be derived nondestructively given its total height (H (m)) and its diameter (d_k (cm)) measured at a relative height (k) along the bole:

$$[1] \quad V_t = [A/(A + 2)]\,[1/(1 - k)]^{2/A}(\pi d_k^2 H/40\,000)$$

Using eq. 1, Forslund estimated the total bole volume of 70 aspen trees (average height 6.8 m) from a single diameter measurement (d_k) on each bole. His results were as accurate as those he achieved by measuring diameter at seven positions (1-m sections) along each bole and deriving volume using Smalian's formula. He found that the best position for measuring d_k was at the average centre of gravity of the bole, for which $k = 0.3$, and he suggested that this position may provide a unique reference location for estimating bole volume, a suggestion also made by Ueno (1978) for estimating stand volume. Forslund further suggested that

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the paracone model may apply not only to aspen but also to all species.

The ability of the paracone model to predict merchantable volume has not been tested. The height (H_m) to the merchantable limit (d_m) on the paracone can be calculated using the following equation:

$$[2] \quad H_m = [1 - (1 - 0.3)(d_m/d_{0.3})^A]H$$

The volume (V_m) below the level of H_m at which diameter is d_m can be derived from eq. 1 as follows:

$$[3] \quad V_m = V_t - [A/(A + 2)(\pi d_m^2(H - H_m))/40\,000]$$

Wood *et al.* (1990) developed a modification of importance sampling, which they call the "centroid method." The centroid method, requiring only one diameter measurement at a calculated relative height, along with stump, merchantable, and total height measurements, also emphasizes the importance of the relative height of 0.3 for estimating total volume of mature tree boles, many of which approximate frusta of second-degree paraboloids. For such solid shapes, the relative height of the centroid (h_c) is 0.2929 (Wood *et al.* 1990), i.e., half the volume occurs both above and below this position; a relative height of 0.3, then, is a convenient compromise. Using the reasonable assumption that the cross-sectional area of the bole of a mature tree decreases linearly with height (Gray 1956), and formulae developed by Gregoire *et al.* (1986), volume (V (m^3)) outside or inside bark, depending on whether cross-sectional area (A_c) at h_c is outside or inside bark, is estimated as

$$[4] \quad V = Q/2[A_c/(H - h_c)]/10\,000$$

with

$$[5] \quad Q = 2H(H_u - H_l) + (H_l^2 - H_u^2)$$

where

H is total tree height
H_u is the height to a point on the upper bole (as merchantable or total height)
H_l is the height to a point on the lower bole (as stump height)
A_c is the cross-sectional area at height h_c

$$[6] \quad h_c = H - [(H - H_l)^2 - 0.5Q]^{0.5}$$

where

h_c is the height to the centroid position on a truncated paraboloid.

Recent research in Australia has demonstrated that the centroid method is accurate for estimating the volume of plantation stands of radiata pine (*Pinus radiata* D. Don) (Wood *et al.* 1990) and the bole volume of various commercial Australian hardwoods (Wood and Wiant 1990).

The purpose of this study on selected Australian and American tree species was to compare estimates of bole volume derived nondestructively using the centroid and paracone methods.

Methods

The four data sets used comprised a total of 957 sample trees measured for diameter inside bark at short intervals along the bole and were as follows: 114 felled trees of radiata pine (average interval between points of measurement approximately 1 m); 140 felled trees of six Australian hardwoods (average interval 2.3 m); 538 standing Appalachian hardwoods (American hardwoods, A) measured using either a tripod-mounted Spiegel Relaskop or Tele-Relaskop (average interval 1.2 m); and 165 felled Appalachian hardwoods (American hardwoods, B) (average interval 1.2 m) (Table 1). Using these measurements, volumes were derived for each short section of each tree using Smalian's formula and then accumulated to give estimates of "true" bole volume. Linearly interpolated values were used to estimate appropriate heights and diameters for the centroid and paracone methods. Total volumes inside bark were calculated for all trees. Merchantable volumes inside bark were calculated for trees that had at least a 2.4 m bole length to the merchantable top, but stems with fewer than four diameter measurements along the bole and those for which diameters were not measured beyond half height were excluded from the merchantable volume calculations.

The results were compared using a modification of Freese's (1960) test of accuracy (Rennie and Wiant 1978). The P%-test statistic is the lower confidence bound for the upper $1 - \alpha$ quantile of the absolute percent differences ($\alpha = 0.05$) (Reynolds 1984).

Results and discussion

The results indicate that the paracone method predicts total volume slightly better than the centroid method for all data sets, except the radiata pine (Table 2). Notice that the mean total volume estimates for the paracone method are consistently smaller by approximately 2.5% than the corresponding centroid estimates. The reason for this can be seen by examining the volume equations. The paracone model for total volume using the diameter at the 0.3 position reduces to

$$V_t = 0.689\pi\, d_{0.3}^2 H/40\,000$$

The centroid method reduces to a similar equation for total volume. Using eq. 6 with $H_l = 0$, $H_u = H$, and $Q = H^2$, the position for diameter measurement is $h_c = 0.293H$, which is the centroid of a paraboloid. Equation 4 then reduces to

$$V_t = 0.707\pi\, d_{0.293}^2 H/40\,000$$

Since the relative heights of 0.3 and 0.293 are very close, the centroid model for total volume is essentially the same as the paracone model, except for the constant. Therefore, as expected, the centroid method yields slightly larger total volume estimates than the paracone method. The overestimate using the centroid method can be an advantage if the stems are generally paraboloidal in form, as is the case with the radiata pine data set; however, if the stems are conical, as perhaps in the smaller hardwoods (American hardwoods, B), the total volume is overestimated more than if the paracone is used.

The merchantable volume estimates were consistently better for the centroid method, and these estimates improved for the smaller hardwoods (American hardwoods, B) as the merchantable height decreased (Table 2). The paracone method generally predicted merchantable volume with accuracy similar to that obtained for total volume, but had a tendency to produce poorer estimates as the merchantable height decreased.

The improved accuracy in merchantable volume estimates using the centroid method as compared with the paracone method is expected for several reasons. First, for deriving merchantable volume the centroid method requires diameter measurements at two positions along the bole, whereas the paracone method requires one only, at the 0.3 position, and the height to the merchantable limit is interpolated from the paracone profile equation [2]. Second, the paracone model is an average profile between the conical and paraboloidal extremes of stem form and was introduced for

TABLE 1. Descriptive statistics of data sets

Data set	No. of trees	Diam. measurements per tree[a]	dbhob (cm)[b] Mean	dbhob (cm)[b] SD	Total ht. (m) Mean	Total ht. (m) SD
Radiata pine	114	30	27	6.7	21	3.1
Australian hardwoods[c]	140	9	72	29.7	36	9.8
American hardwoods						
A[d]	538	21	33	10.7	26	4.8
B[e]	165	10	28	10.1	20	3.2

[a]Average number of diameter measurements made on tree boles.
[b]dbhob, diameter at breast height over bark.
[c]Number of trees by species: *Eucalyptus regnans* F. Muell., 22; *Eucalyptus sieberi* L. Johnson, 35; *Eucalyptus diversicolor* F. Muell., 17; *Eucalyptus maculata* Hook., 21; *Eucalyptus pilularis* Smith, 27; *Flindersia brayleyana* F. Muell., 18.
[d]Number of trees by species: *Quercus rubra* L., 120; *Quercus alba* L., 148; *Liriodendron tulipifera* L., 147; *Acer rubrum* L., 123.
[e]Number of trees by species: *Q. rubra*, 16; *Quercus coccinea* Muenchh., 18; *Quercus velutina* Lam., 18; *Q. alba*, 18; *Quercus prinus* L., 18; *Carya* sp., 15; *Liriodendron tulipifera*, 17; *Prunus serotina* Ehrh., 18; *A. rubrum*, 17; *Betula lenta* L., 2; *Betula alleghaniensis* Britt., 1; *Magnolia acuminata* L., 4; *Populus grandidentata* Michx., 3.

TABLE 2. Comparison of mean volumes (m^3) by Smalian, centroid, and paracone methods

	No. of trees	Smalian, mean	Centroid Mean	Centroid P%[a]	Centroid % diff.[b]	Paracone Mean	Paracone P%[a]	Paracone % diff.[b]	Paraboloid Mean	Paraboloid P%[a]	Paraboloid % diff.[b]
Radiata pine											
Total vol.	114	0.573	0.547	11.8	−4.6	0.533	15.3	−7.0	0.552	10.5	−3.6
Merchantable vol.											
To 10-cm dib	114	0.557	0.531	12.1	−4.6	0.516	16.5	−7.4	0.537	10.8	−3.5
To 12-cm dib	114	0.541	0.515	13.0	−4.9	0.500	17.5	−7.6	0.521	12.2	−3.6
To 14-cm dib	114	0.517	0.492	12.4	−4.9	0.476	18.7	−8.0	0.495	16.8	−4.3
Australian hardwoods											
Total vol.	104	6.930	7.080	16.1	+2.2	6.904	14.9	−0.4			
Merchantable vol.											
To 10-cm dib	71	6.409	6.246	14.2	−2.6	6.069	15.3	−5.3			
To 12-cm dib	71	6.397	6.214	15.1	−2.9	6.059	15.7	−5.3			
To 14-cm dib	69	6.565	6.355	14.1	−3.2	6.220	15.5	−5.3			
American hardwoods, A											
Total vol.	538	1.191	1.231	17.1	+3.3	1.200	16.4	+0.8			
Merchantable vol.											
To 10-cm dib	305	1.171	1.167	15.4	−0.4	1.157	16.8	−1.2			
To 12-cm dib	305	1.155	1.144	15.4	−0.9	1.143	17.9	−1.0			
To 14-cm dib	303	1.140	1.123	15.4	−1.5	1.129	21.6	−1.0			
American hardwoods, B											
Total vol.	165	0.512	0.564	22.8	+10.2	0.550	19.3	+7.4			
Merchantable vol.											
To 10-cm dib	137	0.535	0.562	14.3	+5.0	0.572	19.3	+6.8			
To 12-cm dib	137	0.521	0.539	12.6	+3.4	0.557	19.5	+6.8			
To 14-cm dib	132	0.519	0.531	11.2	+2.3	0.553	21.9	+6.5			

NOTE: dib, diameter inside bark.
[a]Lower confidence bound for 0.95 quantile of the absolute percent differences.
[b]Compared with Smalian estimates.

simplicity. A visual examination of the profiles for radiata pine indicates that their form is essentially paraboloidal; therefore, the centroid method, which assumes paraboloidal form as used here (other forms may be used, see Wood *et al.* 1990), naturally provides better estimates. If the paracone ($A = 1.5$) is replaced with a paraboloid ($A = 2$) in eqs. 1–3 for the radiata pine data set, the estimates improve substantially (refer to Table 2, last column). However, similar to the paracone model, there is still a tendency for the paraboloid model to be poorer as the merchantable height decreases.

Examination of predictions for individual trees indicates that much of the variation for the paracone method (as shown by the P%-value in Table 2) is due to errors in estimating the merchantable volume of the smaller trees. Merchantable volume estimates for large trees are generally

similar for both methods. A possible explanation for this variation is that the paracone model overestimates the volume of the conical top section and underestimates the lower section, which usually has some basal flare. Hence, for total volume estimates, the errors in the two regions will tend to compensate. When considering merchantable volume, however, the top section is subtracted from the total. Hence, as the merchantable height decreases, the paracone model will increasingly underestimate the volume of the lower section, since it will subtract too much for the upper section. Because only the smaller stems are thus affected, the mean volumes for the paracone method in Table 2 remain relatively stable, whereas the P%-values generally increase when smaller trees are present in the sample.

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